Pascack Valley Regional High School District

Pascack Hills High School, Montvale, New Jersey
Pascack Valley High School, Hillsdale, New Jersey

Course Name: **Algebra II w/ Trigonometry Enhanced (MA4400)**
Algebra II w/ Trigonometry (MA4300/4400/4500)
Honors Algebra II w/ Trigonometry (MA4600)

Born On: August, 2016
Previous Revision: August, 2020
Current Revision: August, 2023
Board Approval: 8/28/23

COURSE DESCRIPTION: Algebra II w/ Trigonometry/Enhanced/Honors

Algebra II w/ Trigonometry Enhanced (MA4400), *Algebra II w/ Trigonometry* (MA4300/4500), and *Honors Algebra II w/ Trigonometry* (MA4600) are full year college preparatory, five-credit courses, aligned with the *New Jersey Student Learning Standards,* that build on students' prior work with linear, quadratic, and exponential functions and extend their repertoire of functions to include polynomial, rational, and radical functions. Students work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. All three courses emphasize deep conceptual understanding, but *Algebra II w/ Trigonometry Enhanced* (MA4400) and *Honors Algebra II w/ Trigonometry* (MA4600) place additional emphasis on procedural fluency and cover several plus standards (denoted below with an *) that relate to function inverses, logarithms, polynomials, complex numbers, rational expressions, the unit circle, trigonometric functions, and trigonometric identities. In addition, *Honors Algebra 2 w/ Trigonometry (*MA4600) goes into greater depth with rational and trigonometric functions, and students are expected to solve more challenging, non-routine problems.

All mathematics courses in the Pascack Valley Regional High School District *are* designed to address multiple learning styles and needs, and accommodations and modifications are made for students with disabilities, multilingual students, students at risk of failure, gifted and talented students, and students with 504 plans. *Algebra II w/ Trigonometry* and *Honors Algebra II w/ Trigonometry* build on concepts learned and skills developed in *Geometry* and *Algebra I*, while also spiraling in those concepts and skills to reinforce and strengthen students' algebraic and geometric foundation. Additionally, *Algebra II w/ Trigonometry* anticipates higher-level mathematics that will be learned in *Precalculus* and beyond, and enrichment opportunities are provided to challenge students and engage them in rich, interesting mathematics. Students are encouraged to analyze data using tools and models to make valid and reliable claims (9.4.12.IML.3), and various technologies are integrated throughout the curriculum, including graphing calculators, specialized software, and various Internet programs and subscriptions. These tools enrich the curriculum by giving students' access to additional mathematical representations, and they also help to differentiate by providing students with additional options to engage with mathematical tasks.

The Pascack Valley Regional High School Mathematics Department integrates 21st century life and career skills across its courses, with the dual goal of informing students about careers and fields of study that use mathematics (9.3.ST.5, 9.3.ST-ET.5 and 9.3.ST-SM.2), and helping students improve the quantitative, mathematical, and statistical reasoning skills they will need to be effective producers and consumers of quantitative information in their everyday lives (9.2.12.CAP.2). Mathematics courses address the *New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills*, with a particular emphasis on demonstrating the ability to reflect, analyze and use creative skills and ideas (9.4.12.CI.1), investigating new challenges and opportunities for personal growth, advancement and transition (9.4.12.CI.3), identifying problem-solving strategies used in the development of an innovative product or practice (9.4.12.CT.1), and explaining the potential benefits of collaborating to enhance critical thinking and problem solving (9.4.12.CT.2). Mathematics courses also address the *New Jersey Student Learning Standards for English Language Arts Companion Standards*, with a particular focus on following complex multistep procedures (RST.9-10.3/RST.11-12.3),

determining the meaning of symbols, key terms, and other domain-specific words and phrases (RST.9-10.4/RST.11-12.3), and translating quantitative or technical information expressed in words into visual forms and translating information expressed visually or mathematically into words (RST.9-10.7). Similarly, the mathematics department seeks to support students by providing them with opportunities to use quantitative, statistical, and mathematical reasoning in interdisciplinary contexts, in contexts that are meaningful to students, and in contexts that attend to the contributions and perspectives of historically marginalized groups. Specifically, mathematics courses will look to incorporate, when appropriate, contributions and experiences of people from the LGBTQ+ community and individuals with disabilities, and references to issues of social and cultural relevance, including climate change.

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Content/Topic:	**Key Learning Items/Concepts and Pacing Guide**	**Observable Proficiencies and Skills:**	**NJSLS**	**Formative, Summative, Benchmark, and Alternative Assessments**	**Core Instructional and Supplemental Materials/ Modifications and Accommodations**
Unit 1 – Functions **Time:** 4 weeks *(see column 2 for a more detailed breakdown)* **Content Statement:** Students will be able to model real-life phenomena with different types of functions. **Enduring Understandings:** For a function f(x) that has an inverse, the domain/input for f(x) is the inverse function's range/output and that the range/output for f(x) is the inverse function's domain/input. Models can be written as functions. Function domain and range come from the consideration of what inputs are permitted and what outputs will result.	**Key learning items/concepts:** 1. Graphs of Square Roots, Cube Roots, Piecewise-defined functions, Step Functions, and Absolute Value Functions (2 weeks) 2. Function Transformations (1 week) 3. Inverses (1 week) **Content-specific modifications and accommodations** - use multiple representations to support conceptual understanding - use technology to enhance student engagement - utilize graphing technology and patty paper to support understanding of function	*Write expressions in equivalent forms to solve problems.* *Create equations that describe numbers or relationships* *Represent and solve equations and inequalities graphically.* *Understand the concept of a function and use function notation.* *Analyze functions using different representations.* *Build a function that models a relationship between two quantities* *Build new functions from existing*	**NJSLS Content Standards** A-SSE 1, 3 A-CED 1 A-REI 11 F-IF 3-4, 6-9 F-BF 1, 3, 4 *F-BF 4b-d, 5 **NJSLS SMP** MP1. Make sense of problems and persevere in solving them MP2. Construct viable arguments and critique the reasoning of others MP3. Reason abstractly and quantitatively MP4. Model with mathematics MP5. Attend to precision MP6. Use appropriate tools strategically MP7. Look for and make use of structure MP8. Look for and express regularity in repeated reasoning **NJSLS for ELA Companion**	Students will be assessed regularly throughout this course, with a focus on both conceptual understanding and procedural fluency. Assessment tools may include the following: - quizzes (F) - tests (S) - performance tasks (F/S) - projects (S) - homework (F) - discussions (F) - journals (F) - Form A, B, or C benchmark (B) - alternative assessments (A) - graph your name with piecewise functions (F)	Selection of primary sources *Suggestion(s):* Texts: Pearson Algebra II w/ Trigonometry Common Core (on grade level); Larson Algebra II (advanced); Deltamath (remediation, on grade level, and advanced) A.SSE.B.4 Course of Antibiotics F.IF.C.8b Carbon 14 dating in practice I F.IF.C.7e Logistic Growth Model A.REI.D.11 Ideal Gas Law F.BF.A.1b A Sum of Functions F.BF.B.4a Temperatures in degrees Fahrenheit and Celsius **Modifications and Accommodations:** **Students with special needs:** Teachers and support staff will attend to all modifications and accommodations listed in students' IEPs and 504s. Teachers will incorporate manipulatives, extra time, alternative assessments, scaffolding, spiraling, technology, and flexible grouping to support student learning. **Multilingual students:** Teachers and support staff will work to support multilingual students in their first language and in English, providing materials and/or resources to support students'

Algebra II w/ Trigonometry: *Algebra II w/ Trigonometry* builds on students' prior work with linear, quadratic, and exponential functions and extends their repertoire of functions to include polynomial, rational, and radical functions. Students work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms.					
	transformations and inverses **Interdisciplinary/additional connections** - draw on literal equations from science - draw on student interest and experience in modeling real-world phenomena with piecewise functions	*functions.* *Interpret the structure of expressions.* *Represent and solve equations and inequalities graphically.* *Interpret functions that arise in applications in terms of the context.* *Analyze functions using different expressions* *Build a function that models a relationship between two quantities.* *Build new functions from existing functions.* **Find inverse functions.*	**Standards** RST.9-10.3 RST.9-10.4 RST.9-10.7 RST.11-12.3 RST.11-12.4 **NJSLS-CLKS** **- 21st Century Life and Careers** 9.4.12.CI.1 9.4.12.CI.3 9.4.12.CT.1 9.4.12.CT.2 **- Technology** 9.4.12.IML.3 **- Career Education** 9.2.12.CAP.2 9.3.ST.5 9.3.ST-ET.5 9.3.ST-SM.2 **NJSLS – CSDT** 8.1.12.DA.1 8.1.12.DA.5 8.1.12.DA.6 8.1.12.AP.1 8.2.12.ETW.2		understanding. Students will be given additional time, as appropriate, and translation tools will be utilized as needed. **Students at risk of school failure:** Formative and summative data will be used to monitor student success, and students at risk of failure will receive additional supports and services, which may include parent consultation, extra help, and differentiation strategies, including small group instruction, group work, scaffolding, and spiraling. **Gifted and Talented Students:** Students who excel in their mastery of course standards will be further challenged with more complex tasks, extensions of concepts and skills, and extended problem solving and critical thinking opportunities.

Algebra II w/ Trigonometry: *Algebra II w/ Trigonometry* builds on students' prior work with linear, quadratic, and exponential functions and extends their repertoire of functions to include polynomial, rational, and radical functions. Students work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms.

Content/Topic:	**Key Learning Items/Concepts and Pacing Guide**	**Observable Proficiencies and Skills:**	**NJSLS**	**Formative, Summative, Benchmark, and Alternative Assessments**	**Core Instructional and Supplemental Materials/ Modifications and Accommodations**
Unit 2 –Exponentials and Logarithms **Time:** 12 weeks *(see column 2 for a more detailed breakdown)* **Content Statement:** Students will be able to model real-life phenomena with exponential and logarithmic functions. **Enduring Understandings:** Exponents and logarithms have an inverse relationship. Radical notation is a representation of rational exponents. Exponential functions grow differently than linear functions.	**Key learning items/concepts:** 1. Radicals and rational exponents (2 weeks) 2. Graph exponential functions; interpret parameters of exponential functions (4 weeks) 3. Exponential growth and decay (1 week) 4. Derive formula for a finite geometric series (1 week) 5. Logarithms; Solve Exponential Equations with base 2, 10, or e using Logarithms and Technology (4 weeks) **Content-specific modifications and accommodations** - use multiple representations to support conceptual understanding - use technology to	*Extend the properties of exponents to rational exponents.* *Write expressions in equivalent forms to solve problems.* *Analyze functions using different representations.* *Build a function that models a relationship between two quantities* *Construct and compare linear, quadratic, and exponential models and solve problems.* *Interpret expressions for*	**NJSLS Content Standards** N-RN 1, 2 A-SSE 3, 4 F-IF 7, 8 F-BF 2 F-LE 1-5 **NJSLS SMP** MP1. Make sense of problems and persevere in solving them MP2. Construct viable arguments and critique the reasoning of others MP3. Reason abstractly and quantitatively MP4. Model with mathematics MP5. Attend to precision MP6. Use appropriate tools strategically MP7. Look for and make use of structure MP8. Look for and express regularity in repeated reasoning	Students will be assessed regularly throughout this course, with a focus on both conceptual understanding and procedural fluency. Assessment tools may include the following: - quizzes (F) - tests (S) - performance tasks (F/S) - projects (S) - homework (F) - discussions (F) - journals (F) - Form A, B, or C benchmark (B) - alternative assessments (A) - exp/logarithmic exploration (F)	Selection of primary sources *Suggestion(s):* Texts: Pearson Algebra II w/ Trigonometry Common Core (on grade level); Larson Algebra II (advanced); Deltamath (remediation, on grade level, and advanced) F.IF.C.7c Graphs of Power Functions F.IF.B.4, F.IF.C.7e Model air plane acrobatics F.LE.A.4 Carbon 14 dating N.RN.A.1 Evaluating Exponential Expressions N.RN.A.2 Rational or Irrational? A.SSE.B.3c Forms of exponential expressions F.LE.A.2 Rumors F.LE.B.5, F.LE.A.2 Exponential Parameters **Modifications and Accommodations:** **Students with special needs:** Teachers and support staff will attend to all modifications and accommodations listed in students' IEPs and 504s. Teachers will incorporate manipulatives, extra time, alternative assessments, scaffolding, spiraling, technology, and flexible grouping to support student learning.

	enhance student engagement - differentiate problem sets to support different learners **Interdisciplinary/additional connections** - explore exponential and logarithmic applications in the Richter scale, ph scale, compounded interest and climate change - explore population growth for different groups around the world and in the United States	*functions in terms of the situation they model.* *Construct and compare linear, quadratic, and exponential models and solve problems*.	**NJSLS for ELA Companion Standards** RST.9-10.3 RST.9-10.4 RST.9-10.7 RST.11-12.3 RST.11-12.4 **NJSLS-CLKS** **- 21st Century Life and Careers** 9.4.12.CI.1 9.4.12.CI.3 9.4.12.CT.1 9.4.12.CT.2 **- Technology** 9.4.12.IML.3 **- Career Education** 9.2.12.CAP.2 9.3.ST.5 9.3.ST-ET.5 9.3.ST-SM.2 **NJSLS – CSDT** 8.1.12.DA.1 8.1.12.DA.5 8.1.12.DA.6 8.1.12.AP.1 8.2.12.ETW.2		**Multilingual students:** Teachers and support staff will work to support multilingual students in their first language and in English, providing materials and/or resources to support students' understanding. Students will be given additional time, as appropriate, and translation tools will be utilized as needed. **Students at risk of school failure:** Formative and summative data will be used to monitor student success, and students at risk of failure will receive additional supports and services, which may include parent consultation, extra help, and differentiation strategies, including small group instruction, group work, scaffolding, and spiraling. **Gifted and Talented Students:** Students who excel in their mastery of course standards will be further challenged with more complex tasks, extensions of concepts and skills, and extended problem solving and critical thinking opportunities.

Algebra II w/ Trigonometry: *Algebra II w/ Trigonometry* builds on students' prior work with linear, quadratic, and exponential functions and extends their repertoire of functions to include polynomial, rational, and radical functions. Students work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms.

Content/Topic:	**Key Learning Items/Concepts and Pacing Guide**	**Observable Proficiencies and Skills:**	**NJSLS**	**Formative, Summative, Benchmark, and Alternative Assessments**	**Core Instructional and Supplemental Materials/ Modifications and Accommodations**
Unit 3 – Polynomials and Rationals **Time:** 8 weeks *(see column 2 for a more detailed breakdown)* **Content Statement:** Students will be able to model real-life phenomena with polynomial and rational functions. **Enduring Understandings:** Complex number i is defined such that $i^2 = -1$. For a polynomial $p(x)$ and a number a: - $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$ - $(x - a)$ is a factor of $p(x)$ if and only if $p(a) = 0$ Factors of polynomials can be used to identify zeros to be used to develop a rough graph of the polynomial function.	**Key learning items/concepts:** 1. Operations with polynomials; complex Numbers; Solve quadratics with complex solutions (2 weeks) 2. graph polynomials; polynomial identities; polynomial division; remainder theorem (3 weeks) 3. solve simple rational and radical equations (2 weeks) 4. Systems of three linear equations (1 week) **Content-specific modifications and accommodations** - use multiple representations to support conceptual understanding - use technology to enhance student engagement	*Use properties of rational and irrational numbers.* *Perform arithmetic operations with complex numbers.* *Use complex numbers in polynomial identities and equations.* *Interpret the structure of expressions.* *Perform arithmetic operations on polynomials* *Use polynomial identities to solve problems.* *Rewrite rational*	**NJSLS Content Standards** N-RN 3 N-CN 1, 2, 7 A-SSE 1, 2 A-APR 1-4, 6 A-REI 2, 6, 7, 11 F-IF 7 *N-CN 4-6, 8, 9 *A-APR 7 **NJSLS SMP** MP1. Make sense of problems and persevere in solving them MP2. Construct viable arguments and critique the reasoning of others MP3. Reason abstractly and quantitatively MP4. Model with mathematics MP5. Attend to precision MP6. Use appropriate tools strategically MP7. Look for and make use of structure MP8. Look for and express regularity in repeated reasoning	Students will be assessed regularly throughout this course, with a focus on both conceptual understanding and procedural fluency. Assessment tools may include the following: - quizzes (F) - tests (S) - performance tasks (F/S) - projects (S) - homework (F) - discussions (F) - journals (F) - Form A, B, or C benchmark (B) - alternative assessments (A) - polynomial graphing task (F)	Selection of primary sources *Suggestion(s):* Texts: Pearson Algebra II w/ Trigonometry Common Core (on grade level); Larson Algebra II (advanced); Deltamath (remediation, on grade level, and advanced) A.REI.C.7 Linear and Quadratic System A.REI.C.6 Pairs of Whole Numbers A.REI.A.2 Radical Equations A.APR.B.3 Graphing from Factors III A.APR.C.4 Trina's Triangles A.APR.D.6 Combined Fuel Efficiency A.SSE.A.2 A Cubic Identity A.REI.A.1 Products and Reciprocals N.CN.A.1 Complex number patterns N.CN.A.2 Powers of a complex number N.CN.C.7, A.REI.B.4b Completing the square **Modifications and Accommodations:** **Students with special needs:**

Inverse relationships exist between roots and powers.	- use technology to support students' understanding of systems of three linear equations **Interdisciplinary/additional connections** - explore some of the theorems involving polynomials, including their historical contexts	*expressions.* *Understand solving equations as a process of reasoning and explain the reasoning.* *Represent and solve equations and inequalities graphically.* *Analyze functions using different representations.* **Represent complex numbers and their operations on the complex plane.*	**NJSLS for ELA Companion Standards** RST.9-10.3 RST.9-10.4 RST.9-10.7 RST.11-12.3 RST.11-12.4 **NJSLS-CLKS** **- 21st Century Life and Careers** 9.4.12.CI.1 9.4.12.CI.3 9.4.12.CT.1 9.4.12.CT.2 **- Technology** 9.4.12.IML.3 **- Career Education** 9.2.12.CAP.2 9.3.ST.5 9.3.ST-ET.5 9.3.ST-SM.2 **NJSLS – CSDT** 8.1.12.DA.1 8.1.12.DA.5 8.1.12.DA.6 8.1.12.AP.1 8.2.12.ETW.2		Teachers and support staff will attend to all modifications and accommodations listed in students' IEPs and 504s. Teachers will incorporate manipulatives, extra time, alternative assessments, scaffolding, spiraling, technology, and flexible grouping to support student learning. **Multilingual students:** Teachers and support staff will work to support multilingual students in their first language and in English, providing materials and/or resources to support students' understanding. Students will be given additional time, as appropriate, and translation tools will be utilized as needed. **Students at risk of school failure:** Formative and summative data will be used to monitor student success, and students at risk of failure will receive additional supports and services, which may include parent consultation, extra help, and differentiation strategies, including small group instruction, group work, scaffolding, and spiraling. **Gifted and Talented Students:** Students who excel in their mastery of course standards will be further challenged with more complex tasks, extensions of concepts and skills, and extended problem solving and critical thinking opportunities.

Algebra II w/ Trigonometry: *Algebra II w/ Trigonometry* builds on students' prior work with linear, quadratic, and exponential functions and extends their repertoire of functions to include polynomial, rational, and radical functions. Students work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms.

Content/Topic:	**Key Learning Items/Concepts and Pacing Guide**	**Observable Proficiencies and Skills:**	**NJSLS**	**Formative, Summative, Benchmark, and Alternative Assessments**	**Core Instructional and Supplemental Materials/ Modifications and Accommodations**
Unit 4 – Trigonometric Functions **Time:** 8 weeks *(see column 2 for a more detailed breakdown)* **Content Statement:** Students will be able to model real-life phenomena with trigonometric functions. **Enduring Understandings:** The radian measure of an angle is defined as the length of the arc on the unit circle that is subtended by the angle. The unit circle enables us to determine the characteristics of the trigonometric functions.	**Key learning items/concepts:** 1. Unit circle; radians (3 weeks) 2. Pythagorean identities (1 week) 3. Graph trig functions (period, midline, amplitude) (3 weeks) **Content-specific modifications and accommodations** - use multiple representations and technology to support conceptual understanding - use Geometer's Sketchpad to animate the Unit Circle, and unwrap the unit circle to reinforce key aspects of sine and cosine functinos **Interdisciplinary/additional connections**	*Analyze functions using different representations.* *Extend the domain of trigonometric functions using the unit circle.* *Model periodic phenomena with trigonometric functions.* *Prove and apply trigonometric identities.*	**NJSLS Content Standards** F-IF 7 F-TF 1, 2, 5, 8 *F-TF 3, 4, 6, 7 **NJSLS SMP** MP1. Make sense of problems and persevere in solving them MP2. Construct viable arguments and critique the reasoning of others MP3. Reason abstractly and quantitatively MP4. Model with mathematics MP5. Attend to precision MP6. Use appropriate tools strategically MP7. Look for and make use of structure MP8. Look for and express regularity in repeated reasoning **NJSLS for ELA Companion Standards** RST.9-10.3 RST.9-10.4	Students will be assessed regularly throughout this course, with a focus on both conceptual understanding and procedural fluency. Assessment tools may include the following: - quizzes (F) - tests (S) - performance tasks (F/S) - projects (S) - homework (F) - discussions (F) - journals (F) - Form A, B, or C benchmark (B) - alternative assessments	Selection of primary sources *Suggestion(s):* Texts: Pearson Algebra II w/ Trigonometry Common Core (on grade level); Larson Algebra II (advanced); Deltamath (remediation, on grade level, and advanced) F.TF.A.1 Bicycle Wheel F.TF.A.2 What exactly is a radian? F.TF.A.2 Trigonometric functions for arbitrary angles (radians) F.TF.A.2 Trig Functions and the Unit Circle F.TF.B.5 As the Wheel Turns F.TF.C.8 Trigonometric Ratios and the Pythagorean Theorem F.IF.C.9 Throwing Baseballs **Modifications and Accommodations:** **Students with special needs:** Teachers and support staff will attend to all modifications and accommodations listed in students' IEPs and 504s. Teachers will

	- connect trig functions to sinusoidal phenomena, such as temperature, daylight, or position on a Ferris Wheel		RST.9-10.7 RST.11-12.3 RST.11-12.4 **NJSLS-CLKS** **- 21st Century Life and Careers** 9.4.12.CI.1 9.4.12.CI.3 9.4.12.CT.1 9.4.12.CT.2 **- Technology** 9.4.12.IML.3 **- Career Education** 9.2.12.CAP.2 9.3.ST.5 9.3.ST-ET.5 9.3.ST-SM.2 **NJSLS – CSDT** 8.1.12.DA.1 8.1.12.DA.5 8.1.12.DA.6 8.1.12.AP.1 8.2.12.ETW.2	(A) - trig modeling project (S)	incorporate manipulatives, extra time, alternative assessments, scaffolding, spiraling, technology, and flexible grouping to support student learning. **Multilingual students:** Teachers and support staff will work to support multilingual students in their first language and in English, providing materials and/or resources to support students' understanding. Students will be given additional time, as appropriate, and translation tools will be utilized as needed. **Students at risk of school failure:** Formative and summative data will be used to monitor student success, and students at risk of failure will receive additional supports and services, which may include parent consultation, extra help, and differentiation strategies, including small group instruction, group work, scaffolding, and spiraling. **Gifted and Talented Students:** Students who excel in their mastery of course standards will be further challenged with more complex tasks, extensions of concepts and skills, and extended problem solving and critical thinking opportunities.